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FLUID RESISTANCE TO CYLINDERS IN ACCELERATED MOTION

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(Proc. Paper 1113)

SYNOPSIS

The resistance to cylinders accelerated from rest in water was determined experimentally. The cylinders were of various length-diameter ratios and were accelerated vertically with constant drive forces. A correlation between coefficient of resistance and an acceleration modulus using Reynolds number and the length-diameter ratio as parameters was found to exist.[2]

Resistance of real fluids to accelerating cylinders has not been determined except for the special case in which a real fluid approaches the characteristics of an ideal fluid and the resistance is determined by means of the added mass concept.

Dimensional analysis was employed in this study to determine the relationship between variables in the case of real fluid resistance. The following dimensionless groups were formulated: NR, Reynolds number; Na, acceleration modulus; $N\dot{a}$, acceleration rate change modulus.

The experimental arrangement including a dynamic system with a kinematic value recorder resembled an Attwood's machine. Lateral displacement of the cylinder was restrained during acceleration.

It was determined that fluid resistance to cylinders accelerated from rest could be expressed in terms of the uniform motion resistance relationship,

$$R = \frac{1}{2} C \rho A v^2$$

for which an appropriate coefficient of resistance,

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2. Prepared from a thesis presented to the University of California in partial fulfillment of the requirements for the degree of Master of Science.

$$C = f(NR, Na, geometry)$$

for acceleration can be found experimentally.

INTRODUCTION

Knowledge concerning the total fluid resistance to a cylinder in uniform motion has been well established on a phenomenological basis by Relf(1) and Wieselsberger(2) in 1914 and 1923. Their analysis was based upon Newton's earlier resistance concept as modified by the application of the then recently developed technique of dimensional analysis. The observed phenomena have been reasonably interpreted by the use of the boundary layer theory in real fluids and substantiated by experimentation concerned with the pressure and velocity distribution around the cylinder and its wake.(3)

Resistance to objects in non-uniform motion was observed experimentally in 1786 by Du Buat.(4) A reduction of the expected acceleration of the objects in water was observed, which he explained by attributing an increased mass to the objects. Green(5) and Stokes(6) formulated an equation which appears to substantiate the physical concept of an added or virtual mass. However, this equation is based upon the conditions of an ideal fluid which rejects the possibility of fluid resistance. Further, the equation expresses the resistance as a function only of the geometry of the object and the density of the fluid.

With the advent of aerodynamics practical application of the knowledge concerning fluid resistance became active. In the case of non-uniform motion such as the ascension of lighter-than-air craft and vibrating ships in water, experimenters and mathematicians accepted the concept of added mass and determined added mass factors for various shapes. Reference(7) summarizes the investigations on added mass. Fortuitously, their results were applied to conditions which approximated the ideal fluid. Thus, there was an indication of experimental and theoretical correlation. Inconsistencies did result, however.

These inconsistencies and the concept of a constant added mass were questioned by a few, among whom were investigators for the Aeronautical Research Council of Britain in 1914.(8,9) They were familiar with the success in the dimensional analysis approach which aided the solution of resistance problems in uniform motion. Using a more general case of dimensional analysis they concluded that total fluid resistance in non-uniform motion as in uniform motion would not only be a function of the geometry of the body and the characteristics of the fluid, but also a function of the state of fluid motion. The concept was reasonably demonstrated qualitatively by experiment but apparently not pursued. This can be attributed partly to the serious difficulties which are encountered in the experimental reproduction of a non-uniform state of motion and the measurement of the variables. Further, the concept of added mass provides a more convenient experimental approach, and values of resistance in real fluid under conditions approaching an ideal fluid state have been required and are useful. Experimenters continue to develop methods for the determination of added mass in potential flow.(10,11)

In 1949 and 1951, Iversen and Balent,(7) resuming the investigation of resistance in non-uniform motion based upon the dimensional analysis

approach and the fluid resistance concept, published quantitative experimental results of fluid resistance of a disc accelerated from rest. Their results appeared to correlate the factors indicated by a dimensional analysis except the Reynolds number. The Reynolds number range in the test was above 10^3, the range in uniform motion in which there is no variation in resistance with Reynolds number variation. This was a significant step in the generalization of the special case of uniform motion which has been successfully solved by Relf, Wieselsberger and others using the same concepts.

Concurrently a unique problem developed which concerned the forces on coastal structures (primarily cylindrical piles) induced by wave motion. The fluid motion in this case has been shown to be oscillatory in nature approximating simple harmonic motion.

Experimenters with the problem attempted to interpret observed data by defining the total resistance at any time in terms of the uniform motion drag obtained from instantaneous velocity conditions added linearly to the added mass effect obtained from the instantaneous acceleration condition.(12) This interpretation approximated the observed data. However, in view of the previously cited observations there is a question as to whether the resistance in the case of uniform motion can be generalized to include resistance in the case of non-uniform motion by linearly adding an accelerative effect.

This study constitutes a continuation of the investigation of resistance in non-uniform motion based upon the correlating factors indicated in a dimensional analysis and the fluid resistance concept.

Fundamental Considerations

For the case of uniform motion, total resistance to a cylinder is defined as a combination of viscous shear and pressure. These components have been investigated and interpreted.(3) For real fluid flow in the turbulent state over an object viscosity will create a velocity gradient in a layer at the object surface which is described as a boundary layer. These layers are of extremely small thickness. However, they are not only the reason for the frictional resistance, but also affect the entire flow pattern around the cylinder. The flow pattern controls the pressure resistance. Pressure resistance results from the unbalanced summation of pressure over the cylinder surface and is dependent upon the formation of a wake behind the object.

Since the cylinder geometry and Reynolds number appear to contain the observable variables which describe these mechanisms of resistance, resistance is described as a function of cylinder geometry and Reynolds number of the flow. In this case of uniform motion the cylinder velocity and diameter if used in the Reynolds number, describe the flow.

At very low Reynolds numbers in uniform motion ($N_R < 1$), there is no apparent wake. The area affected by frictional resistance is the entire surface of the cylinder, so frictional resistance dominates. As the Reynolds numbers increase, the laminar layer is forced to separate, a wake develops and weak eddies are seen to form. A fully developed wake occurs near Reynolds number 250. Friction and pressure resistance are then of similar magnitude. Between Reynolds numbers of 10^3 and 10^5 the wake and resistance remain fairly constant. Near Reynolds number of 10^5 the familiar turbulent boundary layer shift to the rear reduces the wake and the resistance, which in this case was predominantly pressure.(3,13,18)

In the case of non-uniform motion in the form of acceleration it has been shown qualitatively that the resistance mechanisms observable in the boundary layers and wake formations are present as in uniform motion.(3) Two types of acceleration conditions were observed. They were the impulsive movement of cylinders from rest and the uniform acceleration of cylinders from rest. The only additional effect due to the accelerations noted was that flow conditions in accelerated motion for any Reynolds number were similar to flow conditions in uniform motion at a lower Reynolds number. Therefore, the total resistance during acceleration must be the result of the resistance mechanisms noted in uniform motion combined with the inertia of the fluid in the process of changing the flow pattern regime. The observations indicate that the mechansism cannot be considered separately, but must be considered in mutually effective combination to form a total fluid resistance.

No attempt based upon these qualitative observations has been made to predict the magnitude of resistance which would be measured experimentally. However, Goldstein and Rosenhead(14) and Blasius(15) have determined quantitatively one phase of the time lag of flow pattern development. They approximated analytically the conditions for the boundary layer separation at the threshold of wake formation during acceleration from rest of a cylinder in real fluid.

A phenomenon which occurs in conjunction with the fluid resistance of a cylinder is the oscillation of the side thrust upon the cylinder in a direction away from the last vortex in the Karman vortex street. In this connection it is noted that the effective diameter for use in various relationships is increased by the displacement caused by the thrust. However, a cylinder is stable, for the amplitude of the displacement will not increase with each successive cycle.

An indication of the manner in which variables may effect the resistance for cases of non-uniform motion can be determined best from a dimensional analysis. Using the variables (see NOTATION) which influence fluid resistance to submerged cylinders of a particular geometry (length-diameter ratio) in non-uniform motion at relatively low velocities, it can be shown that the resistance for a unit length of cylinder can be generally stated as:

$$\frac{R}{L} = f\left(\frac{d\rho v}{\mu}, \frac{d a}{v^2}, \frac{d^2 \dot{a}}{v^3}\right)\rho d v^2 \qquad (1)$$

The dimensionless groups which may be an indication of experimental correlation in the case of non-uniform motion are then:

(1) $N_R = \dfrac{d\rho v}{\mu}$, Reynolds Number

(2) $N_a = \dfrac{d a}{v^2}$, Acceleration modulus (this is of the same form as the Froude Number)

(3) $N_{\dot{a}} = \dfrac{d^2 \dot{a}}{v^3}$, Acceleration rate change modulus (this higher order group can be extended indefinitely to $\dfrac{d^{n-1}}{v^n} \times \dfrac{d^n y}{d t^n}$

Equation (1) is recognized as similar in form to the resistance equation utilized for the special case of uniform motion. Adopting this form for non-uniform motion, equation (1) may be written:

$$R = \frac{1}{2} C \rho A v^2 \tag{2}$$

C is defined as the coefficient of resistance where:

$$C = f(N_R, N_a, N_{\dot{a}}, \text{geometry}) \tag{3}$$

and from equation (2)

$$C = \frac{2R}{\rho A v^2}$$

For a generalized non-uniform motion, the higher order acceleration dimensionless groups expressing the rate of change of the motion would be expected to be less effective as the order and the velocity increases. However, it has been clearly demonstrated analytically and qualitatively (14, 15, 3) that at relatively low velocities a body moving in a viscous fluid with any arbitrary motion experiences an interaction between itself and the fluid which at any instance must be affected by the phase of motion preceding the instantaneous conditions. Therefore, widely differing types of fluid flow patterns which exist instantaneously for similar instantaneous variables must be represented by the dimensionless groups in equation (1). The dimensionless groups must be extended to the highest order of acceleration rate change required to describe the motion. No reported experimental observations to determine the magnitude of the effect of these higher order groups have been found.

Because of the difficulties in the determination and presentation of higher order acceleration rate change dimensionless groups, the relationship between resistance coefficient, Reynolds number and acceleration modulus for a specific mode of acceleration was determined in this study. The higher order dimensionless groups were then included in the single parameter which described the mode of acceleration.

Experimental Method

The experimental apparatus was designed to impart rectilinear motion to three cylinders of different geometry and a disc in water in such a manner that at all stages of motion the kinematic and kinetic values of the experimental system and the test objects could be determined.

In the system the object was pulled vertically upward through water in a three foot diameter tank, four feet high, using a falling weight as the driving force. The driving force was suitably directed by means of a tow line with low elastic distortion properties over specially designed low friction, balanced pulleys placed on a frame over the tank. The part of the tow line which was connected to the test object and which ran over the first pulley consisted of a highly flexible stainless steel twisted fishing leader wire, 0.018 inches in

diameter. After passing over the first pulley, the tow wire was connected to a strip of paper one inch wide which in turn after passing over a drum was connected to a pan upon which the drive weights were placed. A continuous time-distance record was traced on the strip of paper by a stylus with a known vibrating frequency. A schematic drawing of the dynamic system showing the constants and variables of the system as operated is shown in Figure 1. The combination recording the towline paper was a special wax surface recorder paper developed for acoustic recording equipment. The stylus which traced the time-distance record on this paper was a one mil phonograph needle connected to a mechanical vibrator actuated by a sine-wave signal generator through an amplifier. The needle was oscillated at fifty cycles per second at right angles to the drum axis and travel direction of the paper. A spring in the adaptor which held the needle provided for the adjustment of the pressure of the needle on the paper. The highly sensitive wax surface made it possible to obtain a good recording with minimum needle pressure. The three foot by four foot tank provided adequate spacing so that side, end and surface interference effects could be neglected based upon the standard criteria developed in towing tanks and wind tunnels.

Three sizes of cylinders were tested; fifteen inches long by one-half inch diameter, fifteen inches long by one inch diameter and five inches long by one inch diameter. All cylinders had buffed polished surfaces. A disc was tested to check the results of this system with those of the previous work of reference.(7)

The lateral motion which accompanies unrestrained cylinders in motion in a fluid was undesirable for purposes of analysis. The cylinders were restrained laterally by means of guide wires. Music wire, 0.040 inches in diameter, was used. A pair of wires at each end of the cylinder was carefully spaced with an allowance of 0.003 inches wider than the cylinder by means of machined lucite spacers. The lucite spacers spaced twenty-one inches vertically were fixed in space by lateral tension lines in the direction of cylinder thrust.

Procedure

Elements in the system which provided friction were the pulleys, the needle on the paper and the cylinder against the guide wires. At the state of impending motion guide wire friction could not be detected. It was possible to eliminate virtually all of the small friction contributed by the pulleys and needles during the experimentation except the difference between static and kinetic friction by balancing the system at the point of impending motion before adding the drive weights.

The reliability of the experimental system was established by comparing the measured acceleration and calculated acceleration of the system when operated with a known drive force without the resistance of water, assuming air resistance to be negligible.

Figure 1 which shows the experimental dynamic system states and defines the kinematic and kinetic values involved in the force equation for the unidirectional accelerated motion of the system. The force equation for the system is:

$$F + w - R = \sum M \cdot a \qquad (5)$$

The fluid resistance to the test object at any instant may be defined therefore as:

$$R = F + w - \sum M \times a \tag{6}$$

All of the necessary data for the evaluation of the fluid resistance and its correlating factors were available in the system constants, the drive force for each run, the fluid constants and the instantaneous kinematic conditions.

The time-distance data presented by the oscillating needle on the paper was reduced directly by the use of a microscopic comparator. The comparator read to a least count of 0.0001 inch.

Instantaneous velocities were calculated by use of the finite difference equation outlined in reference(16) and (17) for use in graphical-numerical differentiation. Instantaneous accelerations were calculated by use of the finite difference equation outlined in the same method.

Since the observed data were read to the nearest 0.0001 inch, "smoothing" of the distance-time and velocity-time curves by graphical means was not possible. However, a cross-plot of calculated velocity and acceleration was easily smoothed to a continuous function. Intercurve similarity aided the smoothing process. Expected accelerations at zero velocity were calculated using the published added mass values of references(10) and (11). Expected velocities at zero acceleration were calculated by use of the published values of uniform drag coefficient in equation (2). These initial and terminal values were plotted and used as a comparison with the experimentally determined accelerations and velocities. Figure 2 is typical of the smoothed curves from which kinematic values were taken for dynamic computations.

Results

Figure 3 presents the results of three runs with different drive forces for the cylinder of length-diameter ratio of thirty. A correlation between resistance coefficient and acceleration modulus exists. The experimental relationship between resistance coefficient and acceleration modulus for the acceleration of discs from rest which was determined by reference (7) is also plotted in Figure 3. The experimental results for the acceleration of a disc from rest produced by this experimental apparatus are seen to check and substantiate the previously determined relationship. This apparatus was on a smaller scale and utilized a different time-distance recording arrangement than the equipment of reference (7).

Arguments presented in Figure 3, which were determined for values in the three runs at equal instantaneous Reynolds numbers, are presented in Figure 4 by a plot of coefficient of resistance against acceleration modulus using Reynolds number as a parameter. According to the observations of Wieselsberger(2) for uniform motion a cylinder of length-diameter ratio thirty exhibits resistance characteristics which approach those of an infinitely long cylinder. In reference to the familiar experimental determination of coefficient of resistance against Reynolds number in uniform motion, a fairly constant value of about 0.95 for coefficient of resistance occurs between Reynolds numbers of 10^3 and 5×10^3. The results plotted in Figures 3 and 4 for each run approach coincidence for instantaneous Reynolds numbers between 1.30 and 4.75×10^3. Prior to approaching this region of constant

resistance coefficients instantaneous Reynolds numbers for non-uniform motion have been shown to indicate flow patterns similar to those in uniform motion described by those for lower instantaneous Reynolds numbers. However, within the region of constant resistance coefficients, the flow pattern appears to achieve eventually the constant condition in which instantaneous Reynolds numbers no longer affect it or its resulting coefficient of resistance. The resistance coefficient shown in Figures 3 and 4 approaches the uniform resistance coefficient of 0.95 as values of velocity increase with a reduction of the acceleration modulus and its affect upon the resistance coefficient.

Figure 5 presents the results of two runs with different drive forces for a cylinder of length-diameter ratio fifteen. Wieselsberger(2) observed that a cylinder of length-diameter ratio fifteen exhibited a resistance coefficient in uniform motion which does not show a marked deviation from the coefficient of resistance in uniform motion of a cylinder of length-diameter ratio thirty. In these two runs no Reynolds number effect was observed because the instantaneous Reynolds numbers approached the values in which the coefficient of resistance is not affected by Reynolds numbers. A comparison with the results of Figure 3 plotted again in Figure 5 indicates that the geometry effect on the coefficient of resistance is similar to that in uniform motion. There is no marked deviation of resistance between cylinders of length-diameter ratio fifteen and thirty.

Figure 6 presents the results of three runs with different drive forces for the cylinder with length-diameter ratio five. However, a cylinder of this length-diameter ratio exhibits a marked reduction of the coefficient of resistance in uniform motion of a cylinder of length-diameter ratio thirty.(2) A fairly constant value of about 0.78 exists between values of Reynolds numbe 10^3 to 10^5. No Reynolds number effect was observed as in the case of the cylinder of length-diameter ratio fifteen, for the instantaneous Reynolds numbers approached the values in which the coefficient of resistance is not affected by Reynolds number. A comparison with the results of Figure 3 plotted again in Figure 6 indicate the geometry effect on the coefficient of resistance is similar to that in uniform motion. There is a marked reduction of the coefficient of resistance for a cylinder of length-diameter ratio five compared to one of length-diameter ratio thirty. No analytical formulation of the results correlated in Figures 5-8 was attempted.

CONCLUSIONS

The following conclusions were based upon the analysis of experimental results and the preceding discussion.

1. Fluid resistance to cylinders in accelerated motion can be expressed in terms of the uniform motion resistance relationship, $R = \frac{1}{2} C \rho A v^2$ for which an appropriate coefficient of resistance for accelerated motion can be determined.

2. For a particular mode of acceleration, the coefficient of resistance was found to be a function of Reynolds number, acceleration modulus and cylinder geometry.

3. Experimental determination of the resistance coefficient is required since no analytical relationship was found.

4. The resistance concept which describes a unique total resistance for each fluid pattern around an object moving in general motion appears to be sound and is recommended as the basis for further investigation of resistance in non-uniform motion.

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Professors J. W. Johnson, M. ASCE, and A.D.K. Laird contributed significantly in encouraging the execution of the study and the publication of this paper.

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NOTATION

a Instantaneous acceleration - in/sec^2

$\dot{a}$ Instantaneous rate of acceleration change - in/sec^3

A Frontal area of test object - in^2

C Coefficient of resistance

d Diameter of cylinder or disc - in

F Experimental drive force - gram weight

g Acceleration of gravity

L Length of cylinder - in

M Mass of experimental component - gram mass

ΣM Total effective mass of experimental system - gram mass

Na Acceleration modulus - $\frac{da}{v^2}$

$N\dot{a}$ Acceleration rate change modulus - $\frac{d^2\dot{a}}{v^3}$

NR Reynolds number - $\frac{d\rho v}{\mu}$

R Fluid resistance to test objects - grams

v Instantaneous velocity - in/sec

y Distance traveled by test object from origin - in

ρ Density of fluid

μ Absolute viscosity of fluid

BIBLIOGRAPHY

1. Relf, E. F., Discussion of the results of measurements on the resistance of wires, with some additional tests on the resistance of wires of small diameter, AERONAUTICAL RESEARCH COMMITTEE R.M. 102, Gt. Britain, March, 1914.

2. Wieselsberger, C., ERGEBNISSE DER AERODYNAMISCHEN VERSUCHSANSTALT ZU GOTTENGEN, Vol. II, P. 24, 1923.

3. Goldstein, S., MODERN DEVELOPMENTS IN FLUID DYNAMICS, Vol. I and II, pp. 401-440 and pp. 50-88, Oxford, 1938.

4. Du Buat, C., PRINCIPES D'HYDRAULIC, Paris, 1786.

5. Green, G., MATHEMATICAL PAPERS, p. 315, 1833.

6. Stokes, G., MATHEMATICAL AND PHYSICAL PAPERS, Vol. I, p. 17, 1843.

7. Iversen, H. W., and Balent, R., A correlating modulus for fluid resistance in accelerated motion, JOURNAL OF APPLIED PHYSICS, Vol. 22, No. 3, March, 1951. (A summary of "Virtual Mass of Disks" Master's Thesis, University of California, 1949).

8. Fraser, R. A. and Simmons, V. A., The dependence of the resistance of bodies upon acceleration, AERONAUTICAL RESEARCH COMMITTEE R.M. 590, January, 1919.

9. Cowley, W., and Levy, H., On the effect of accelerations on the resistance of a body, AERONAUTICAL RESEARCH COMMITTEE R.M. 612, May, 1918.

10. Rogers, D. A., and Shakshober, M. C., An investigation of the virtual mass of a cylinder vibrating in water, Master's Thesis, Massachusetts Institute of Technology, May, 1953.

11. Stelson, J. M. and Mavis, F. T., Virtual mass and acceleration in fluids, PROCEEDINGS, AMERICAN SOCIETY OF CIVIL ENGINEERS. Vol. 81, No. 670, 1955.

12. Morison, J. R., Johnson, J. W., and O'Brien, M. P., Experimental studies of forces on piles, Proceedings, FOURTH CONFERENCE ON COASTAL ENGINEERING, COUNCIL ON WAVE RESEARCH, Chap. 24, Berkeley, 1954.

13. Prandtl, L., and Tietjens, G. G., APPLIED HYDRO- AND AEROMECHANICS, pp. 279-292, McGraw-Hill Book Co., 1934.

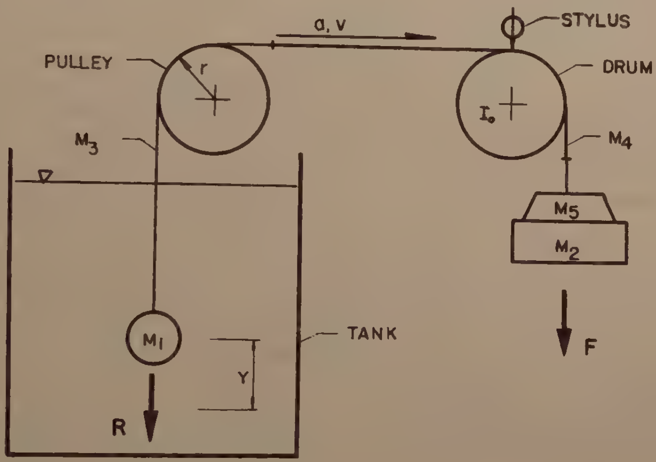
14. Goldstein, S. and Rosenhead, L., Boundary layer growth, PROCEEDINGS, CAMBRIDGE PHILOSOPHICAL SOCIETY, Vol. 32, Pt. 3, 1936.

15. Blasius, ZEITSCHRIFT FUR MATHEMATIK UND PHYSIK, Vol. 56, 1908.

16. Levens, A. S., GRAPHICS IN RESEARCH, Institute of Engineering Research, University of California, Report Issue 6-50, June 9, 1950.

17. Felkel, E. O., DETERMINATION OF ACCELERATION FROM DISPLACEMENT-TIME DATA, Institute of Engineering Research, University of California, Series 11, Issue 16, September, 1951.

18. Rouse, H., ENGINEERING HYDRAULICS, pp. 115-135, John Wiley & Sons, Inc., New York, 1949.



SCHEMATIC OF EXPERIMENTAL APPARATUS

INSTANTANEOUS KINEMATIC VALUES

a = System acceleration v = System velocity y = Distance from origin

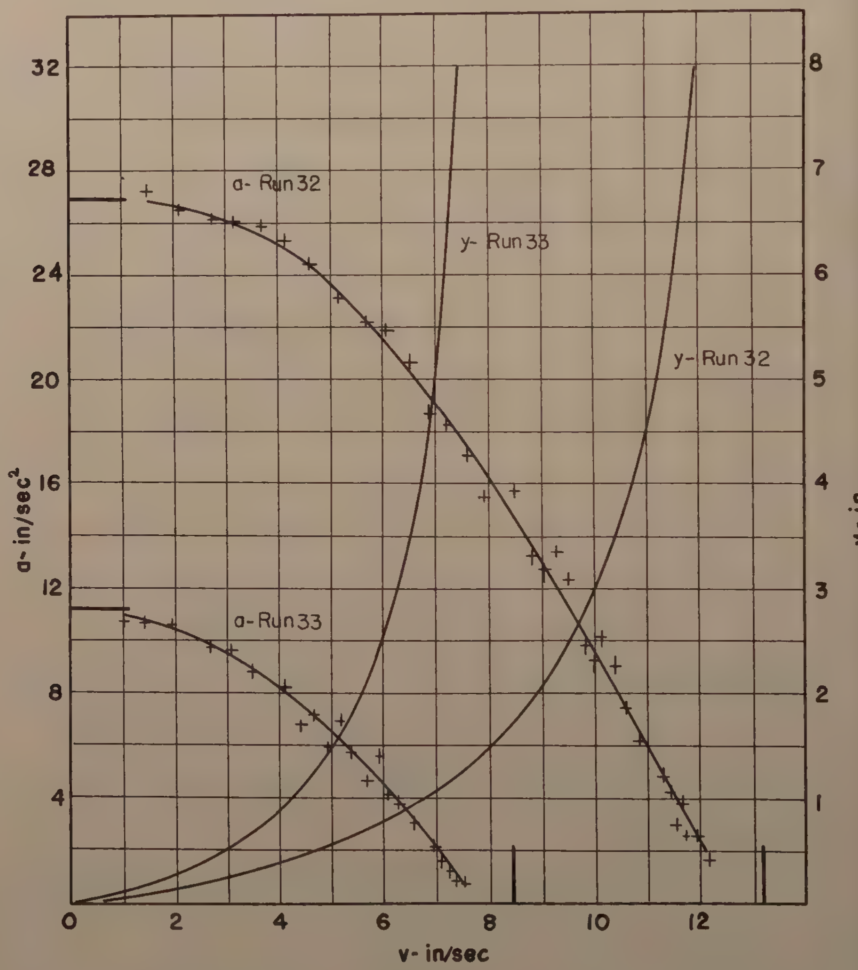
INSTANTANEOUS RESULTANT FORCE ON THE SYSTEM = F + w -R

$F = M_2 \times g$ = Drive force
R = Fluid resistance
$w = k \times y$ = Added drive force from unbalanced towline
where: k = weight of towline plus paper per unit length less buoyancy of submerged wire (negligible)

MASS OF SYSTEM = $\sum M = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7$

M_1 = Mass of test object
M_2 = Drive force weight mass
M_3 = Mass of tow wire
M_4 = Mass of recording paper
M_5 = Mass of static balance at Impending motion
M_6 = Effective mass of pulley = I_o/r^2
M_7 = Effective mass of drum = I_o/r^2
where: I_o = Moment of inertia
r = Radius

FIGURE 1

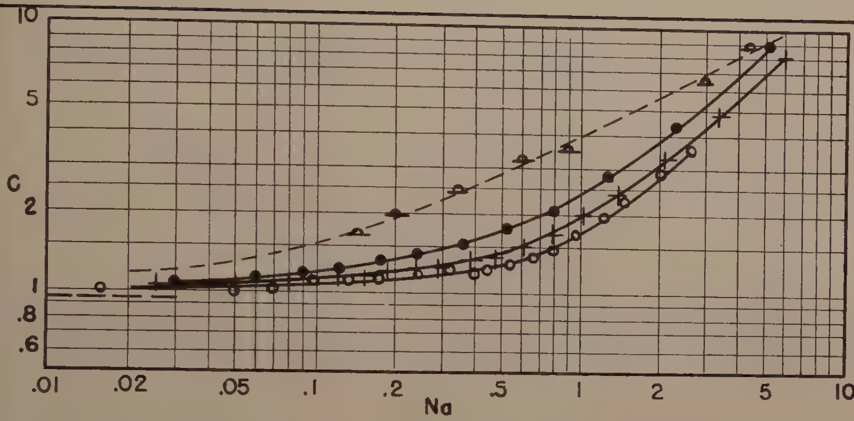


TYPICAL KINEMATIC CONDITIONS

FOR CYLINDERS ACCELERATED FROM REST

RUNS 32,33 — CYLINDER-L/d = 30, d = 0.5 IN.

FIGURE 2



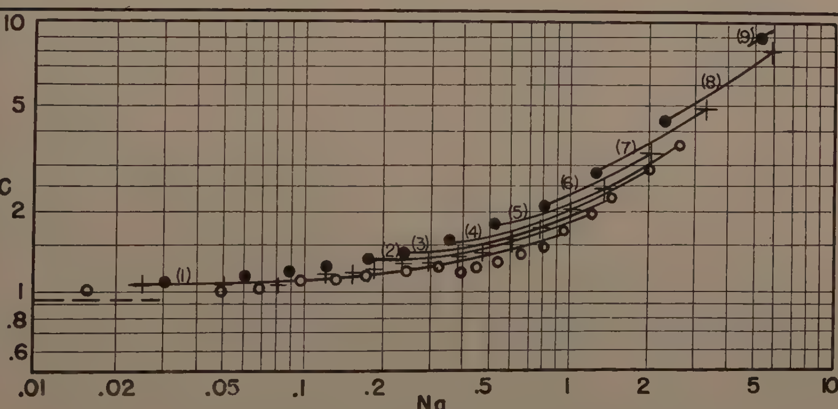
ACCELERATION MODULUS vs COEFFICIENT OF RESISTANCE

FOR CYLINDER – L/d = 30, d = 0.5 IN., ACCELERATED FROM REST BY VARIOUS DRIVE FORCES (F)

RUN 31 ○ F = 50 GM
RUN 32 + F = 25 GM
RUN 33 ● F = 10 GM

RUN D ⏏ (DISC, d = 4.75 IN.)
DATA FOR DISC FROM REFERENCE (7) – – – – – –

FIGURE 3



ACCELERATION MODULUS vs COEFFICIENT OF RESISTANCE

FOR CONSTANT INSTANTANEOUS REYNOLDS NUMBER (NR)

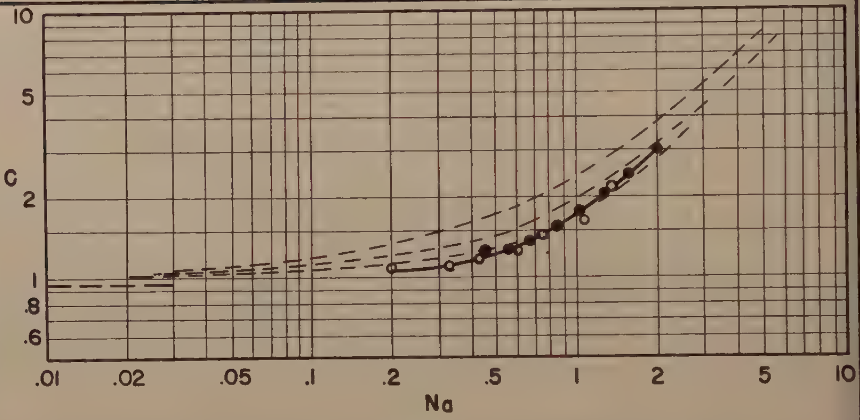
FOR CYLINDER – L/d = 30, d = 0.5 IN., ACCELERATED FROM REST

(1) NR = 1.47 TO 4.75 X 10^3
(2) NR = 1.32 X 10^3
(3) NR = 1.18 X 10^3
RUN 31 ○

(4) NR = 1.03 X 10^3
(5) NR = 8.8 X 10^2
(6) NR = 7.4 X 10^2
RUN 32 +

(7) NR = 5.9 X 10^2
(8) NR = 4.4 X 10^2
(9) NR = 3.0 X 10^2
RUN 33 ●

FIGURE 4



ACCELERATION MODULUS vs COEFFICIENT OF RESISTANCE

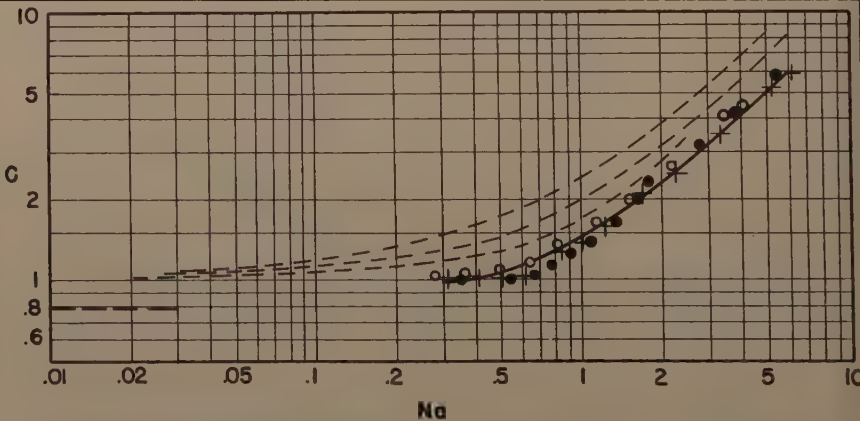
FOR CYLINDER-L/d=15, d=1 IN., ACCELERATED FROM REST

$NR = 2.0 \text{ TO } 4.4 \times 10^3$

RUN 41 ○ F = 25 GM.
RUN 42 ● F = 50 GM.

RUNS 31, 32, 33 — — —
(SEE FIGURE 3)

FIGURE 5



ACCELERATION MODULUS vs COEFFICIENT OF RESISTANCE

FOR CYLINDER-L/d=5, d=1 IN., ACCELERATED FROM REST

$NR = 1.18 \text{ TO } 5.30 \times 10^3$

RUN 51 ○ F = 10 GM.
RUN 52 + F = 15 GM.
RUN 53 ● F = 25 GM.

RUNS 31, 32, 33 — — —
(SEE FIGURE 3)

FIGURE 6

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GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES

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(Proc. Paper 1114)

Tables of functions for the purpose of calculating water-surface profiles in open channels of uniform cross section have been prepared by Bakhmeteff.(1) In deriving these functions he assumed that the conveyance K and the critical discharge Q_c are both exponential functions of the depth variable y with the same exponent n/2 in each case. A wide range of exponents are included in his tables.

Unfortunately, these functions have not been used to the extent that one might expect, largely because of the labor involved in the method outlined by Bakhmeteff. The purpose of this paper is to present a graphical method which greatly reduces the labor involved in using them. A distinct advantage is that the graphical method presented herein permits the direct solution for the depth of water at the end of a reach of prescribed length, whereas in the older methods a series of trial depths had to be assumed until one was found that yielded the required length.

Bakhmeteff's basic variables are η and B, where η is the non-dimensional stage variable

$$\eta = \frac{y}{y_o} \; ; \tag{1}$$

y and y_o are the actual depth and the normal depth, respectively; and B is a function of η defined as

$$B(\eta) = -\int_0^\eta \frac{d\eta}{\eta^n - 1} \tag{2}$$

The equation to be used in computing backwater profiles is

$$L_{12} = \frac{y_o}{S_o}\left[\eta_2 - \eta_1 - (1-\beta)(B_2 - B_1)\right], \tag{3}$$

in which L_{12} is the channel distance between the two cross sections identified by the subscripts 1 and 2, S_o is the slope of the channel bottom and β is a non-dimensional quantity that can be estimated with any of the following formulas

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1. Hydr. Engr., Corps of Engrs., U. S. Dept. of the Army, Mobile, Ala.

$$\beta = \left(\frac{y_c}{y_o}\right)^n \tag{4}$$

$$\beta = \left(\frac{V_o}{V_c}\right)^2 \tag{5}$$

$$\beta = \frac{S_o}{\sigma} \tag{6}$$

In these equations Vo is the normal velocity, V_c the critical velocity, y_o the normal depth, y_c the critical depth, and σ the critical slope. Equation (4) is often used in the determination of β. However, it should be realized that the number β will usually vary a little with the depth y and accordingly an average value for the range of depths under consideration will have to be used. The Bakhmeteff hydraulic exponent n can be determined by plotting y as abscissa and K as ordinate on logarithmic paper and finding twice the slope of the line.

For the purpose of developing this method, it is convenient to multiply both sides of eq 3 by S_o/Y_o and to let

$$I = \frac{L S_o}{y_o} \tag{7}$$

and

$$w = 1 - \beta, \tag{8}$$

so that eq 3 becomes

$$I_{12} = \eta_2 - \eta_1 - w(B_2 - B_1).$$

This can be rearranged in the form

$$I_{12} = (\eta_2 - wB_2) - (\eta_1 - wB_1). \tag{9}$$

If η and B are plotted as ordinate and abscissa, respectively, in rectangular coordinates, a given value of η -wB will appear in the resulting diagram as a straight line having a slope equal to w. Eq. 9 indicates that the vertical displacement between the two lines representing cross sections 1 and 2 is equal to I_{12}. This is illustrated in fig. 1. It is clear from this diagram that if η_1, w and I_{12} are known, the point 2 and the corresponding depth variable η_2 are readily determined with a pair of draftsman's triangles. If several points on a water-surface profile are desired, the construction shown in fig. 2 can be used. The water-surface profile shown in this figure is an M_1 profile. The corresponding construction for an M_2 and an M_3 profile is shown in fig. 3. It should be noted that critical depth occurs at the point of tangency.

There is no difficulty in applying the method to steep channels. The appropriate construction for an S_2 profile representing flow in a steep channel with control at the upstream end is shown in fig. 4. In steep channels the quantities w and I sometimes reach extreme values with the result that the graphical construction runs off the sheet on which the curves have been drawn. In this case the modified procedure illustrated in fig. 5 can be used. As can be seen in this figure the usual construction which is represented by dashed lines,

runs off the sheet. However, by laying out the distance I_{12}/w horizontally, as shown, the solution is readily obtained.

To determine the elevation z directly it is possible to introduce the non-dimensional elevation number

$$\epsilon = \frac{z}{y_o} \tag{10}$$

The appropriate construction is shown in fig. 6. This is a repetition of the same construction as shown in fig. 2 with the addition of an upper diagram to determine the elevation number graphically. The corresponding elevation z is then obtained by multiplying the values of ϵ so determined by the normal depth y_o.

The graphical construction based on the B function fails in the case of a channel with a horizontal or an adverse bottom slope. However, Bakhmeteff has outlined a solution for horizontal channels and Matzke[2] has published tables that permit the solution of problems for channels of adverse slope.

In Bakhmeteff's solution for a horizontal channel the variable η is replaced by the variable

$$\tau = \frac{-y}{y_c} \tag{11}$$

and the function B by the function

$$J_{(\tau)} = \frac{\tau^{n+1}}{n+1} \tag{12}$$

Eq. 9 is replaced by

$$l_{12} = (\tau_2 - wJ_2) - (\tau_1 - wJ_1) \tag{13}$$

where

$$l = \frac{\sigma L}{y_c} \tag{14}$$

and

$$w = \frac{\sigma}{\sigma_c} \tag{15}$$

where σ_c is the critical slope at the critical depth.

In Matzke's solution for a channel with an adverse bottom slope the function B is replaced by the function

$$B'_{(\eta)} = \int_o^{\eta} \frac{d\eta}{\eta^n + 1} \tag{16}$$

The depth variable η is still defined by eq. 1 but it should be understood the normal depth y_o is defined for flow in the opposite direction, i.e., for flow down the channel slope. Eq. 9 is replaced by

$$I_{12} = (-\eta_2 + w B_2') - (-\eta_1 + w B_1') \tag{17}$$

where I is still defined by eq. 7 and the slope w is defined by

$$w = 1 + \beta \tag{18}$$

Other backwater functions have been published from time to time. Those of Bresse,(3) which duplicate the Bakhmeteff functions for the exponent n = 3 are of earlier date and are well known. Some of the functions are based on the assumption that the conveyance and the critical discharge vary according to different exponents. Among these are the functions of Mononobe;(4) Von Seggern;(5) Lee, Babbitt, and Bauman;(6) and Ven Te Chow.(7) Functions for part-full flow through circular conduits have been published by Keifer and Chu.(8)

It is possible to adapt the graphical solution to all of these functions. The appropriate adaptation for the Von Seggern functions is illustrated in fig. 7. In this connection I is still defined by eq. 7 and the slope w is defined as

$$w = -\beta_o \tag{19}$$

where β_0 is the value of β at normal depth. It should be noted that an auxiliary diagram in the upper part of the figure is required to complete the solution since the depth variable η is not immediate identified in the lower diagram. A similar graphical solution for the Keifer and Chu functions was given by the writer in a discussion of the paper by those authors.

A diagram that is quite helpful in selecting the exponent n for channels with trapezoidal and circular sections has been developed by Barbarossa.(9) This diagram which is self-explanatory is shown in fig. 8.

ACKNOWLEDGMENT

The writer wishes to acknowledge the help received from the following persons in the Corps of Engineers, U. S. Army: A. L. Cochran, Office of the Chief of Engineers; R. L. Irwin, Washington District; and F. B. Campbell, Waterways Experiment Station.

REFERENCES

1. Bakhmeteff, B. A., Hydraulics of Open Channels, McGraw-Hill Book Co., New York, N. Y.

2. Matzke, A. E., Varied flow in Open Channels of Adverse Slope, A.S.C.E. Trans., Vol. 102, 1937.

3. Bresse, J.A.C., Mechanique Applique, v.z., Mallet-Bachelier, Paris, 1860 (Table republished in the following publication: Treatise on Hydraulics by Merriman, 10th Ed., Wiley 1916; Weir Experiments, Coefficients, and Formulas, Water-Supply and Irrigation Paper No. 200, by Robert E. Horton Gov't Printing Office, 1907; and Hydraulics of Steady Flow in Open Channels, by S. H. Woodward and C. J. Posey, Wiley 1941.)

4. Mononobe, N., Backwater and Dropdown Curves for uniform Channels, A.S.C.E. Trans., Vol. 103, 1938.

5. Von Seggern, M. E., Integrating the equation of nonuniform flow, A.S.C.E. Trans., Vol. 115, 1950.

6. Lee, M., Babbitt, H. E., and Baumann, E. R., Gradually Varied Flow in Uniform Channels on Mild Slopes, Bul. Series No. 404, U. of Ill., Engr. Exp. Sta., 1952.

7. Ven Te Chow, Integrating the equation of gradually varied flow, A.S.C.E. Proc., Vol. 81, Paper No. 838, 1955.

8. Keifer, C. J. and Chu, H. H., Backwater Functions by Numerical Integration, A.S.C.E. Trans., Vol. 120, 1955.

9. Barbarossa, N. L., unpublished data.

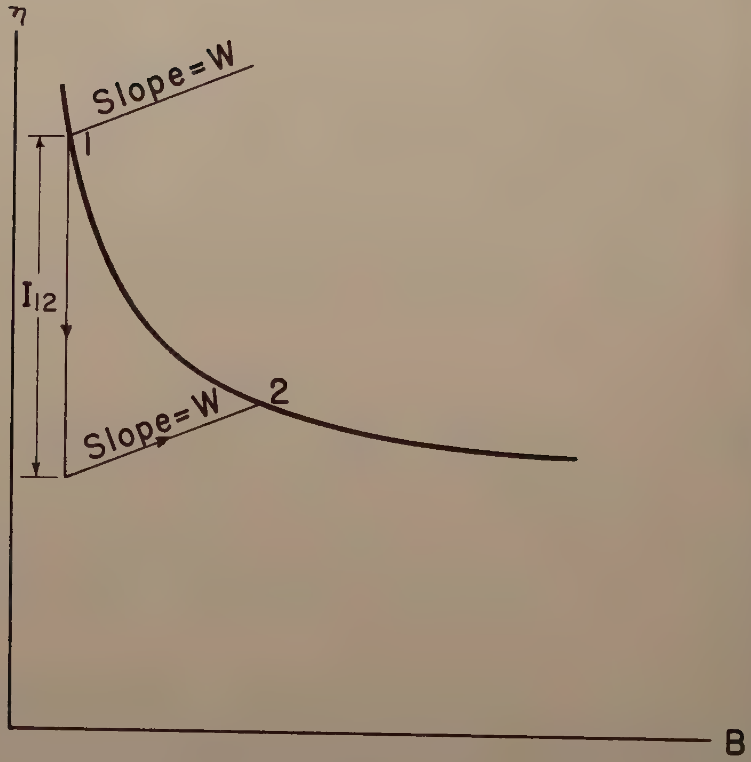


Fig. 1

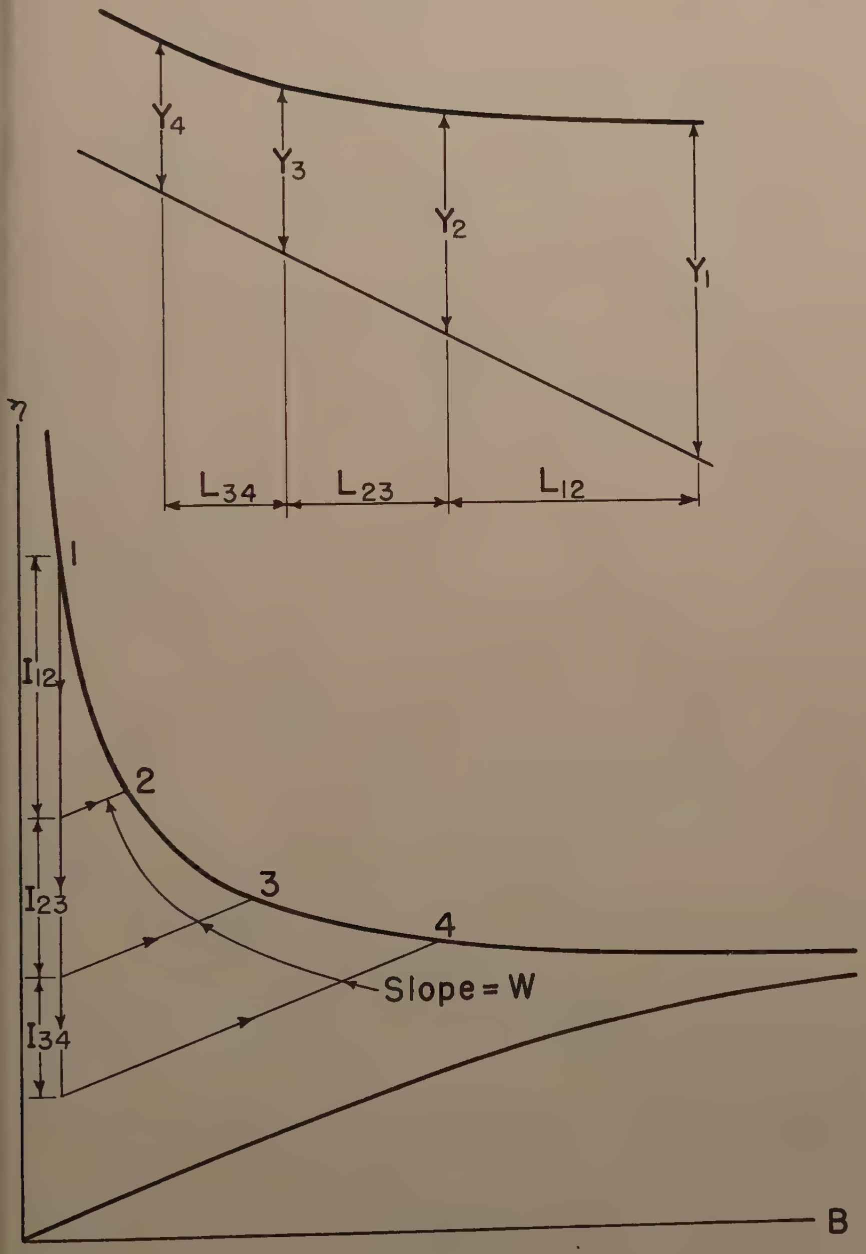


Fig. 2

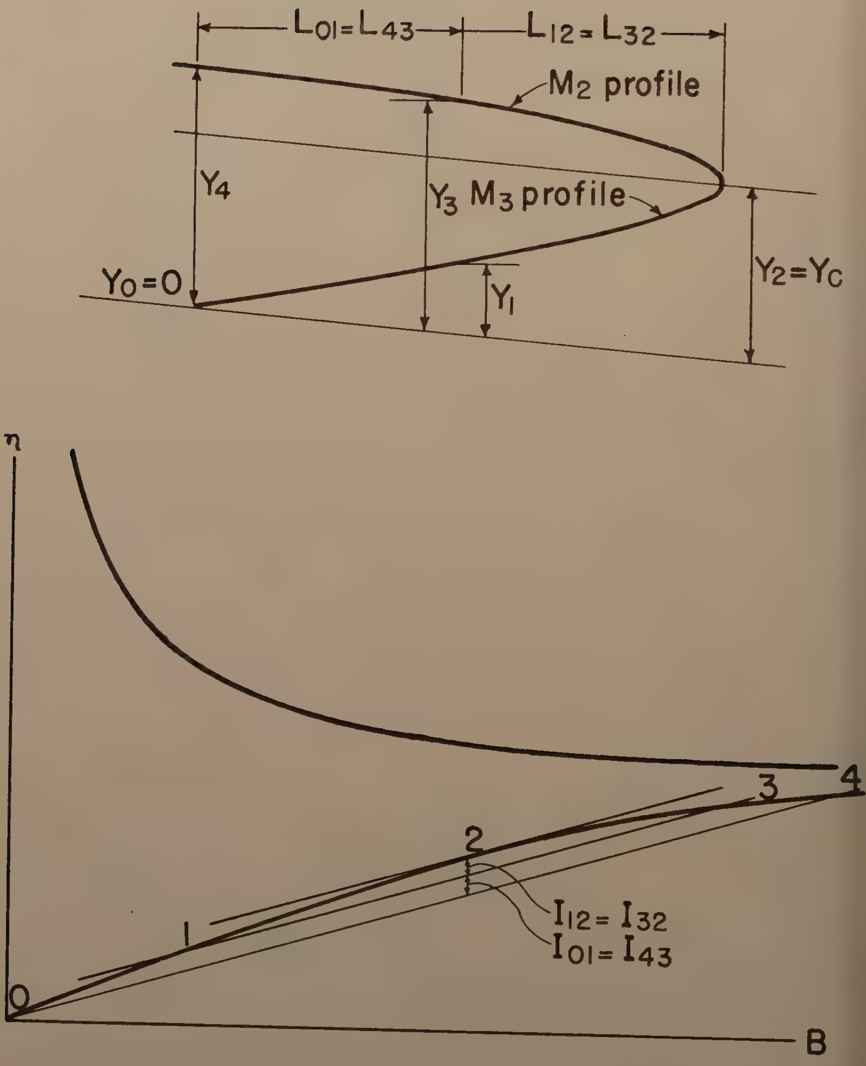

$L_{01}=L_{43}$
$L_{12}=L_{32}$
M_2 profile
Y_4
Y_3
M_3 profile
$Y_0=0$
Y_1
$Y_2=Y_C$
η
0
1
2
3
4
$I_{12}=I_{32}$
$I_{01}=I_{43}$
B

Fig. 3

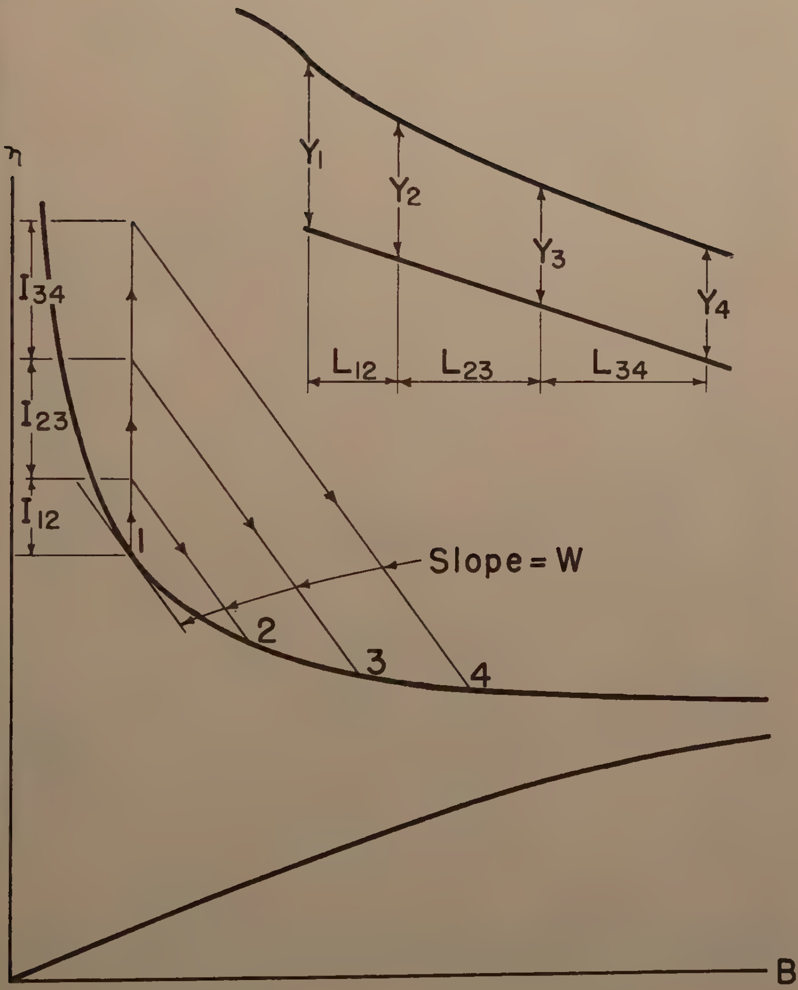


Fig. 4

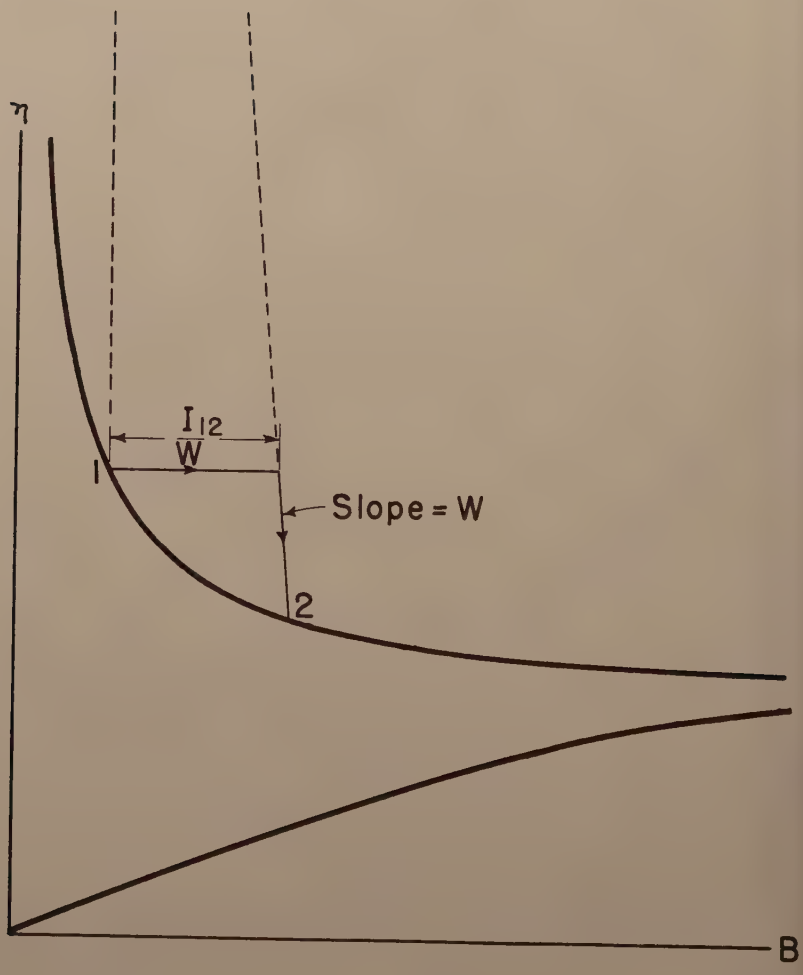


Fig. 5

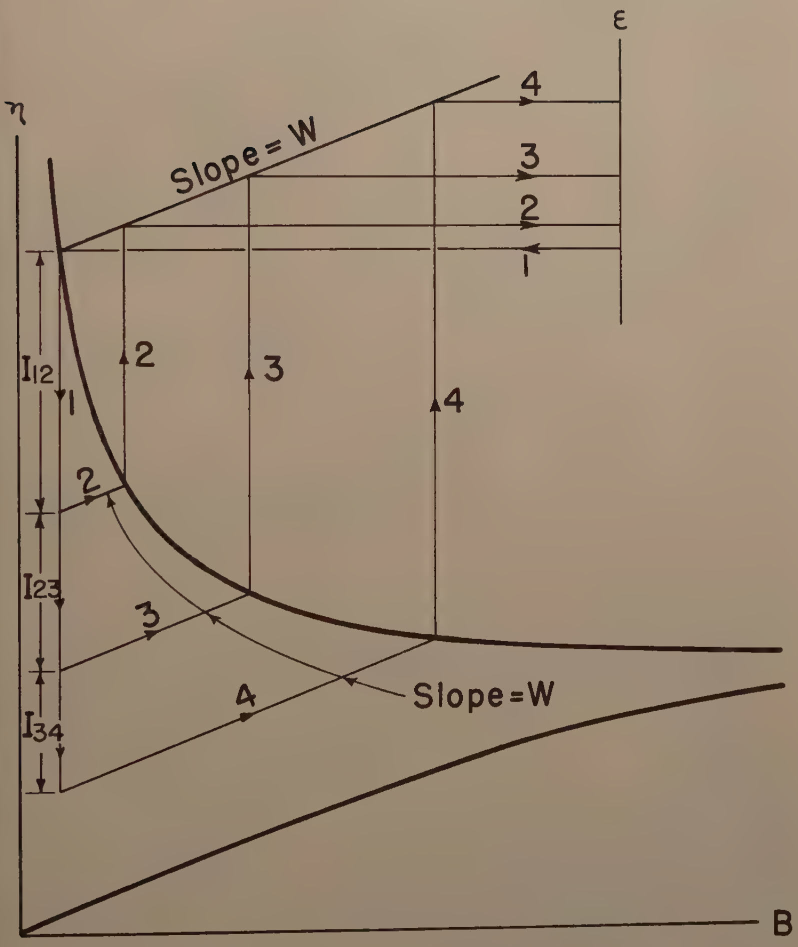
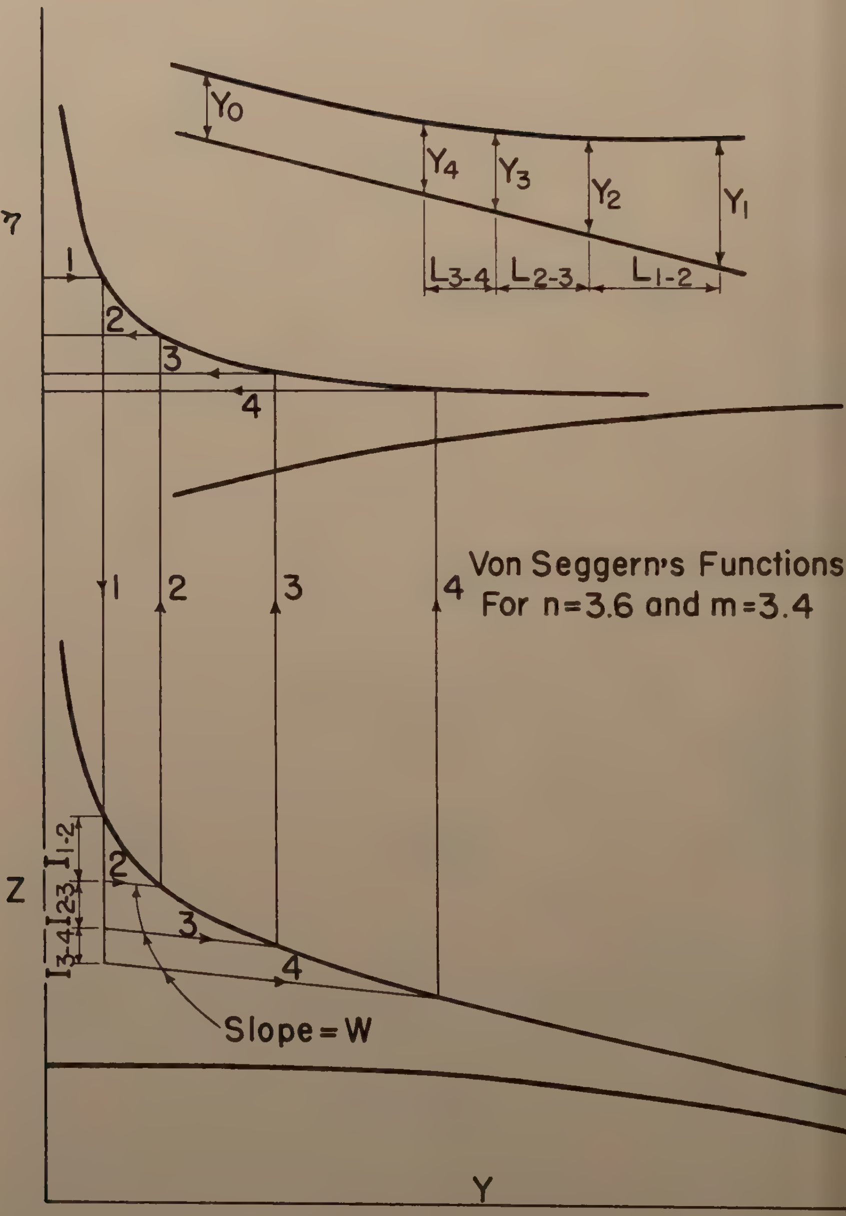
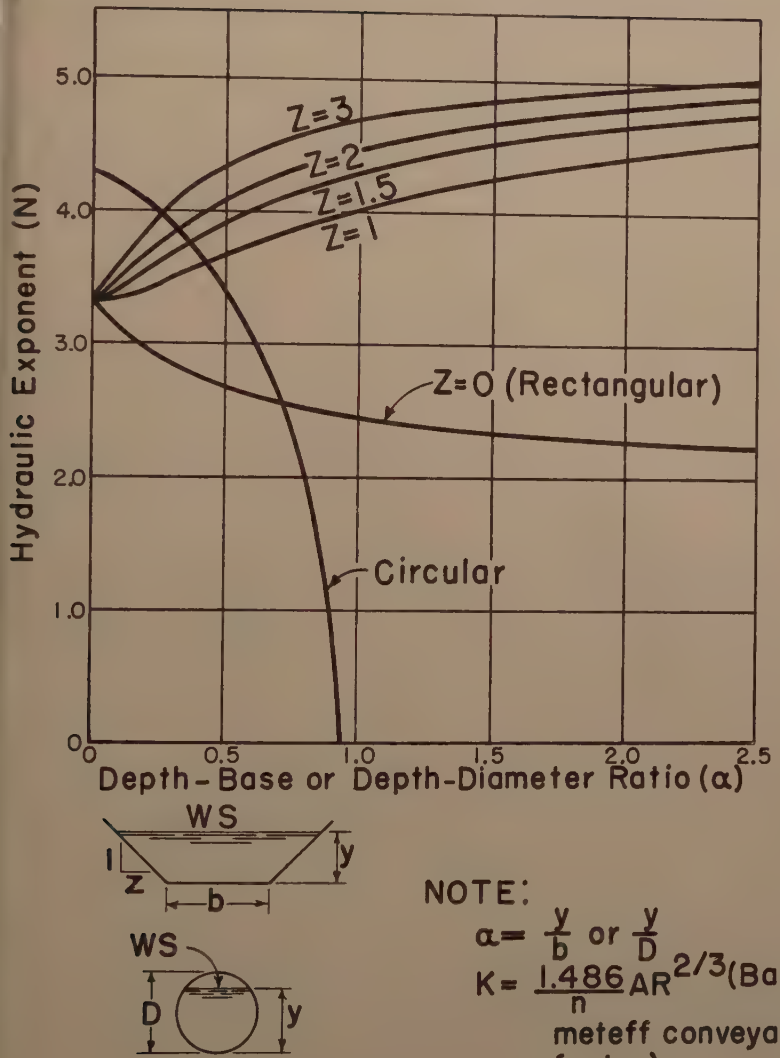


Fig. 6



Y0
Y4
Y3
Y2
Y1
L3-4
L2-3
L1-2
η
1
2
3
4
Von Seggern's Functions
For n=3.6 and m=3.4
Z
I1-2
I2-3
I3-4
Slope=W
Y

Fig. 7



NOTE:

$\alpha = \frac{y}{b}$ or $\frac{y}{D}$

$K = \frac{1.486}{n} AR^{2/3}$ (Bakhmeteff conveyance factor)

OPEN CHANNEL FLOW
HYDRAULIC EXPONENT "N"

Fig. 8

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by Fred W. Blaisdell 1131-5

Note: Paper 1131 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 82, HY 6, December, 1956.

Discussion of "FREE-SURFACE DISTURBANCE ALONG A CHANNEL WALL"

by Amein M. Amein and Melville S. Priest
(Proc. Paper 1005)

CORRECTIONS.—As a result of errors in proofreading made by the printer, several equations in this paper were improperly located. As a result, the section entitled "Analysis" (which starts on page 1005-2) should be rewritten as follows:

Analysis

Quantities pertinent to conditions that are just sufficient for initiation of the free-surface disturbance may be related through the dimensionless parameters of the function

$$\phi(UL/\nu, D/L) = 0 ,$$

where U is mean velocity of flow across the cross-section of initiation, L is length from the inlet crest to the cross-section of initiation, ν is kinematic viscosity of the water, and D is depth of flow at the cross-section of initiation. The relation between the parameters of the above function is shown graphically in Fig. 1. Fitting the experimental data to a power function, the relation may be expressed as

$$UL/\nu = 2{,}750\,(D/L)^{-0.800} \tag{1}$$

This relation is represented by the curve in Fig. 1.

Quantities pertinent to growth of the free-surface disturbance may be related through the dimensionless parameters of the function

$$\psi\,(U\delta/\nu, Ux/\nu) = 0 ,$$

where x and δ are coordinates of points on the outer envelope of the disturbance, the origin of coordinates being at the point of initiation and the x and δ-axes being, respectively, along and normal to the channel wall. Other quantities are as previously defined. The relation between the parameters of the above function is shown graphically in Fig. 2. The advantage in the choice of parameters shown is that, for particular values of U and ν, Fig. 2 indicates the shape of the outer envelope of the disturbance, subject to distortion due to choice of scales. Fitting the experimental data to a power function, the relation may be expressed as

$$U\delta/\nu = 0.00146\,(Ux/\nu)^{1.31} \tag{2}$$

This relation is represented by the curve in Fig. 2. From Eq. 2, other forms such as those commonly used in connection with turbulent boundary layers and wakes may be easily derived. As an example,

$$\delta/x = 0.00686\,(U\delta/\nu)^{0.237}$$

Note: Reprints of the correct version of the entire paper may be obtained from ASCE headquarters.

Discussion of
"FREE OUTLETS AND SELF-PRIMING ACTION OF CULVERTS"

by Wen-Hsiung Li and Calvin C. Patterson
(Proc. Paper 1009)

FRED W. BLAISDELL,[1] M. ASCE.—Messrs. Li and Patterson have presented information on the hydraulics of culverts having square-edged entrances and free outlets. It should be pointed out that the discharge through this type of culvert at any particular time is frequently unpredictable when the inlet is submerged. It is unfortunate that a head-discharge curve was not presented to show this undesirable flow condition. In many instances the head-discharge curve would show that two discharges were possible at the same head for a considerable range of depths over the inlet. The control for the lesser flow would be the entrance acting as an orifice or sluice. The conduit would be full for the greater flow. Under differing field conditions, it is not possible to predetermine which control would exist at any one time or exactly when or at what head the control will change from orifice flow to pipe flow. The authors have determined maximum heads at which the flow changes from orifice or sluice to full pipe, but they have not emphasized that the change is not always made at the same head. Although this head may be determined fairly definitely for a specific laboratory arrangement, small changes in the approach conditions, for example, may result in a greatly different head at which the flow changes from sluice to full pipe. The change from one control to the other and the change in the discharge may occur suddenly. The unpredictability and the magnitude of these changes suggests that performance characteristics of this type are undesirable. This is a major objection to the use of square-edged culvert entrances. These statements are not intended to detract from the facts reported by the authors. They are presented to bring to the attention of others some of the poor performance characteristics of a square-edged culvert entrance and the desirability of giving the entrance the attention it deserves. The authors have performed a valuable service in bringing this information to the attention of the profession.

Even after working on the hydraulics of culverts for 16 years, the writer is being continually surprised and humbled by new developments. He has found that culvert hydraulics are not as simple as he once thought and that there is still much to be learned about the hydraulic performance of culverts. From his own experience and his association with a number of other persons, the writer concludes that the longer one works with culvert hydraulics the more cautious he is in his statements. Conversely, the less experience one has with culvert hydraulics, the more positive his statements are apt to be. It is true that no new laws of culvert hydraulics have been found over this period and that the application of these laws to flow through culverts is not difficult. The difficulty is in determining the type of flow which exists in the

1. Project Supervisor, Agri. Research Service, Soil and Water Conservation Research Branch, U. S. Dept. of Agriculture, St. Anthony Falls Hydra. Lab., Minneapolis, Minn.

culvert so that the proper law can be applied. This requires considerable experience and a thorough knowledge of culvert hydraulics. Summary charts as presented in this paper are an aid to one who thoroughly understands them, but are dangerous in the hands of one who is not familiar with their limitations. Comprehension of all the characteristics of culverts having square-edged entrances is essential before attempting to use the information presented in this or in other papers. Although expressed many times by others, these ideas need occasional re-emphasis.

The curves of Fig. 6, giving orifice or sluice flow for the square-edged entrances, would not exist for what the writer would define as an entrance with good performance characteristics. A good entrance would permit the flow to change directly from weir control for part-full conduits to pipe control for full conduits, with perhaps a small range of heads following neither control when air is sucked in and is carried through the conduit. Again, the authors have presented some good data in Fig. 6 but orifice or sluice control should be "designed out" of a culvert so that a definite head-discharge relationship can be obtained. An alternative would be to design a culvert to eliminate the possibility of the conduit flowing full. However, this is probably a generally less desirable alternative.

The three methods of self-priming illustrated by Messrs. Li and Patterson are familiar to the writer. In addition, priming might occur at the inlet if something were to cause a disturbance there.

The fact that conduit roughness affects the curves of Figs. 7, 8 and 9, as pointed out by the authors, is sufficient reason for suggesting that they be not used for design purposes, contrary to the recommendations of the authors. The difficulty of predicting the horizontal location of the hydraulic jump on flat or gently sloping aprons is well known. In a culvert with a square-edged entrance, a small error in estimating the roughness or a small error in setting the conduit slope can cause a similar considerable shift in the position of the hydraulic jump. This could result in a completely different flow pattern than had been assumed.

The authors use the Froude number when plotting the curve giving the position of the hydraulic grade line at the conduit exit. The Froude number as ordinarily conceived relates to free-surface flow. What is the significance of the Froude number as applied to full closed conduit flow? For example, a Froude number of one in open channel flow designates the critical depth of flow with all its connotations. Does the Froude number as usually defined have some similar significance in respect to the hydraulic grade line in pipes? The writer has observed that the conduit exit ceases to flow completely full when $V/\sqrt{gD}$ is less than about one. Is this related in any way to the minimum energy content of open flow which occurs when the Froude number is one? While it is entirely possible that gravity may influence the position of the hydraulic grade line for an outlet discharging freely, the writer cannot immediately see that the Froude number, as usually conceived, is pertinent to this case. It is likely to lead to considerable confusion if it is necessary to redefine the meaning of the Froude number to fit this special case. The point raised is not with regard to the units used, but with regard to the definition given the terms employed. The ratio $Q/\sqrt{g}\ D^{5/2}$, mentioned by the authors in their introduction, is to be preferred. This ratio can be defined as the dimensionless discharge or the discharge for a 1-ft diameter pipe if Q is in cfs and g is taken as 32.2 ft per sec per sec. It might be pointed out that $Q/\sqrt{g}\ D^{5/2}$ can be obtained by multiplying both the

numerator and the denominator of $V/\sqrt{gD}$ by the pipe area and dividing by a constant. Thus, for circular pipes

$$\frac{V}{\sqrt{gD}} \times \frac{A}{\pi D^2/4} = \frac{Q}{(\pi/4)\sqrt{g}\, D^{5/2}}$$

Another satisfactory variation of the parameter was pointed out to the writer by Alvin G. Anderson, Assistant Professor of Hydraulics at the University of Minnesota. This variation can be defined as a ratio of the velocity head to the pipe diameter. This ratio is one-half the square of the parameter used by the authors or

$$\left(\frac{V}{\sqrt{gD}}\right)^2 \times \frac{1}{2} = \frac{V^2/2g}{D}$$

The use of either of these latter parameters, or defining the term used by the authors as something other than the Froude number, would eliminate the possibility of assuming that the point at which the hydraulic grade line pierces the plane of the outlet is a function of the Froude number as ordinarily conceived.

The curve giving the position of the hydraulic grade line at the plane of the conduit exit is most interesting. It shows that the position of the hydraulic grade line at the conduit exit depends on the velocity (or discharge) and that it is not located at a singular point for all flows. As the authors point out, this is especially important when the head across the culvert is relatively low.

Messrs. Li and Patterson use an indirect method to determine m. They recognize that α_1 and α_2 may have different values. They also recognize that Δ_1 and Δ_2 may be unequal; however, they say ". . . the small differences are of no great practical importance at the present . . ." The writer has obtained values of m over the range of discharges covered by Messrs. Li and Patterson by projecting to the outlet the hydraulic grade line determined in a 110D length of conduit. This method was mentioned by the authors, but apparently was not used by them because it requires longer conduit lengths than they had available in many instances. A comparison of the writer's and the authors' results is thus a comparison of methods as well as a partial evaluation of the approximations mentioned by the authors. The writer's curve parallels that of the authors, but it is slightly lower. In fact, the agreement of the writer's data with the curve of Messrs. Li and Patterson is somewhat better than with the curve obtained by Mr. Rueda,[2] Rueda's curve being slightly higher and flatter.

The pipe slopes used by the writer range between 0 and 36 per cent. Higher relative discharges were obtained than were possible by Messrs. Li and Patterson. Values of m less than 0.5 were obtained at the higher discharges. They were considerably lower for the very high discharges and the steep conduit slope. The writer considers that his curve needs further checking before presenting quantitative results. The point the writer wishes

2. Daniel Rueda-Briceño, "Pressure Conditions at the Outlet of a Pipe," M.S. thesis submitted to the State University of Iowa, February 1954.

to make here is that the curve presented in Fig. 4 is not asymptotic to m = 0.5. The curve as drawn in Fig. 4 well represents the data of Messrs. Li and Patterson, but it should not be extrapolated beyond the data.

While Messrs. Li and Patterson have made a good start in the determination of the point where the hydraulic grade line pierces the plane of a conduit outlet, it is apparent to the writer that much additional work needs to be done

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-ivision sponsorship is indicated by an abbreviation at the end of each Paper Number, the ymbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering echanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), anitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and apping (SU), and Waterways and Harbors (WW) divisions. Papers sponsored by the Board of irection are identified by the symbols (BD). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were ublished in Journals of the various Technical Divisions. To locate papers in the Journals, the ymbols after the paper numbers are followed by a numeral designating the issue of a particular ournal in which the paper appeared. For example, Paper 861 is identified as 861 (SM1) which ndicates that the paper is contained in issue 1 of the Journal of the Soil Mechanics and Foundaions Division.

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CTOBER: 1070(EM4), 1071(EM4), 1072(EM4), 1073(EM4), 1074(HW3), 1075(HW3), 1076(HW3), 1077(HY5), 1078(SA5), 1079(SM4), 1080(SM4), 1081(SM4), 1082(HY5), 1083(SA5), 1084(SA5), 1085(SA5), 1086(PO5), 1087(SA5), 1088(SA5), 1089(SA5), 1090(HW3), 1091(EM4)[c], 1092 (HY5)[c], 1093(HW3)[c], 1094(PO5)[c], 1095(SM4)[c].

OVEMBER: 1096(ST6), 1097(ST6), 1098(ST6), 1099(ST6), 1100(ST6), 1101(ST6), 1102(IR3), 1103 (IR3), 1104(IR3), 1105(IR3), 1106(ST6), 1107(ST6), 1108(ST6), 1109(AT3), 1110(AT3)[c], 1111(IR3)[c], 1112(ST6)[c].

ECEMBER: 1113(HY6), 1114(HY6), 1115(SA6), 1116(SA6), 1117(SU3), 1118(SU3), 1119(WW5), 1120(WW5), 1121(WW5), 1122(WW5), 1123(WW5), 1124(WW5)[c], 1125(BD1)[c], 1126(SA6), 1127 (SA6), 1128(WW5), 1129(SA6)[c], 1130(PO6)[c], 1131(HY6)[c], 1132(PO6), 1133(PO6), 1134(PO6), 1135(BD1).

. Discussion of several papers, grouped by Divisions.

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